

$$\textcircled{1} \quad r(t) = \langle t, 3t - t^2 \rangle \quad t \in [0, 2]$$

$$\quad \quad \quad x(t) \quad y(t)$$

$$\frac{dr}{dt} = \langle 1, 3 - 2t \rangle$$

$$dr = \langle 1, 3 - 2t \rangle dt$$

$$F(x, y) = \langle 3t^2 - t^3, -t^2 \rangle$$

$$\int_0^2 \langle 3t^2 - t^3, -t^2 \rangle \cdot \langle 1, 3 - 2t \rangle dt$$

$$\int_0^2 -t^3 + 2t^3 dt = \int_0^2 t^3 dt = \frac{16}{4} = \boxed{4}$$

$$\textcircled{2} \quad \begin{array}{l} x=t \\ y=\frac{1}{2}t^2 \end{array} \quad t \in [0, 2] \quad \frac{dx}{dt} = 1, \frac{dy}{dt} = t, \quad ds = \sqrt{1+t^2} dt$$

$$2 \int_0^2 \frac{t^3}{t^2} \sqrt{1+t^2} dt = 2 \int_0^2 t \sqrt{1+t^2} dt$$

$$u = 1+t^2$$

$$du = 2t dt$$

$$\int_1^5 \sqrt{u} du = \left[\frac{2}{3} u^{3/2} \right]_1^5$$

$$= \boxed{\frac{2}{3} (5^{3/2} - 1)}$$

$$\begin{aligned} \textcircled{3} \textcircled{a} \quad \frac{\partial^2 f}{\partial x \partial y} &= ax^2 + 3y^2 \\ \frac{\partial^2 f}{\partial y \partial x} &= 6x^2 + by^2 \end{aligned} \Rightarrow \begin{aligned} a &= 6 \\ b &= 3 \end{aligned}$$

$$\begin{aligned} \textcircled{b} \quad &\langle 6x^2y + y^3 + 1, 2x^3 + 3xy^2 + 2 \rangle \\ &f = 2x^3y + xy^3 + x + C \\ &f = 2x^3y + xy^3 + 2y + C \\ &f(x, y) = 2x^3y + xy^3 + x + 2y \end{aligned}$$

$$\textcircled{c} \quad \int_C \vec{F} \cdot d\vec{r}$$

$$C = \{ (e^t \cos t, e^t \sin t) \mid t \in [0, \pi] \}$$

$$\vec{F}(x, y) = \langle 6x^2y + y^3 + 1, 2x^3 + 3y^2 + 2 \rangle$$

$$\vec{r}(t) = \langle e^t \cos t, e^t \sin t \rangle \quad t \in [0, \pi]$$

$$\text{FToLine Integrals} \quad \begin{aligned} \vec{r}(0) &= (1, 0) \\ \vec{r}(\pi) &= (-e^\pi, 0) \end{aligned}$$

$$f(-e^\pi, 0) - f(1, 0) = -e^\pi - 1$$

$$\text{so} \quad \int_C \vec{F} \cdot d\vec{r} = \boxed{-e^\pi - 1}$$

$$\textcircled{4} \quad \vec{F}(x, y, z) = \langle \cos x + 2y^2 + 5yz, 4xy + 5xz, 5xy + 3z^2 \rangle$$

$$f = \sin x + 2xy^2 + 5xyz + C(y, z)$$

$$f = 2xy^2 + 5xyz + C(x, z)$$

$$f = 5xyz + z^3 + C(x, y)$$

$$\boxed{f = \sin x + z^3 + 2xy^2 + 5xyz}$$

$$\textcircled{5} \quad f = xy^2 - y \sin x + y$$

$$r(t=1) = (0, 2) \quad \text{and} \quad r(t=2) = (6, 5)$$

$$f(0, 2) = 2 \quad \text{and} \quad f(6, 5) = 155 - 5 \sin 6$$

$$\text{so by FTOLine Integrals,} \quad \int_C \vec{F} \cdot d\vec{r} = \boxed{153 - 5 \sin 6}$$

⑥

$$C = \{ (10 \cos t, 10 \sin t) \mid t \in [0, 2\pi] \}$$

$$\oint_C (2xy + e^x) dx + (x^2 - \sin y + 3x) dy$$

Green's
Theorem

$$\iint_D (2\cancel{x} + 3) - (2\cancel{x}) dA = 3 \iint_D dA = \boxed{300\pi}$$

⑦

$$C \text{ is } \{ (x, y) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \}$$

$$F(x, y) = \left\langle \frac{x}{a^2}, \frac{y}{b^2} \right\rangle$$

F is conservative

because for $f = \frac{x^2}{2a^2} + \frac{y^2}{2b^2}$, $\nabla f = F$

C is closed so

$$\oint_C \vec{F} \cdot d\vec{r} = \boxed{0}$$

⑧ a

$$\oint_C y^2 dx + 4xy dy = \int_{C_1} y^2 dx + 4xy dy + \int_{C_2} y^2 dx$$

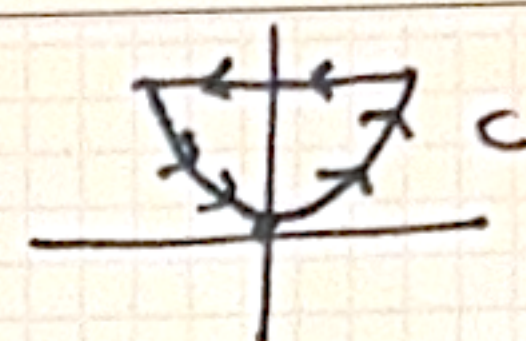
$$C = C_1 \cup C_2$$

$$C_1 = \{ (t, t^2) \mid t \in [-1, 1] \} \quad C_2 = \{ (t, 1) \mid t \in [1, -1] \}$$

$$= \int_{-1}^1 t^4 dt + 4(t)(t^2)(2t) dt + \int_1^{-1} dt \quad \leftarrow \begin{array}{l} \text{runs} \\ \text{backward} \end{array}$$

$$= \int_{-1}^1 t^4 + 8t^4 - 1 dt$$

$$= \int_{-1}^1 9t^4 - 1 dt = \boxed{\frac{8}{5}}$$



$$\textcircled{8} \textcircled{b} \oint_c y^2 dx + 4xy dy$$

$$= \iint_D 4y - 2y dA = \iint_D 2y dA$$

$$= \int_{y=0}^{y=1} \int_{x=-\sqrt{y}}^{x=\sqrt{y}} 2y dx dy$$

$$= \int_0^1 2y \int_{-\sqrt{y}}^{\sqrt{y}} dx dy$$

$$= \int_0^1 4y\sqrt{y} dy = \boxed{\frac{8}{5}}$$

$$\textcircled{9} F = \langle y, z, -2x \rangle$$

$$C \in \{(x, y, z) \mid x^2 + y + z = 1\}$$

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & -2x \end{vmatrix} = \langle -1, 2, -1 \rangle$$

$$\oint_c F \cdot dr = \iint_S (\nabla \times F) \cdot ds$$

Stoke's theorem

$$z = 1 - x - y$$

$$\vec{r}(x, y) = \langle x, y, 1 - x - y \rangle$$

$$\vec{r}_x = \langle 1, 0, -1 \rangle \text{ and } \vec{r}_y = \langle 0, 1, -1 \rangle$$

$$ds = \vec{r}_x \times \vec{r}_y = \langle 1, 1, 1 \rangle$$

$$\iint_S \langle -1, 2, -1 \rangle \cdot \langle 1, 1, 1 \rangle dx dy = \iint_S 0 dx dy = \boxed{0}$$

(10)

$$\iint_{\Sigma} \vec{F} \cdot \vec{n} \, dS = \iiint_V \nabla \cdot \vec{F} \, dV$$

$$\text{where } V = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$$

$$\nabla \cdot \vec{F} = 3x^2 + 3y^2 + 3z^2$$
$$\int_{x=-1}^x=1 \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \int_{z=-\sqrt{1-x^2-y^2}}^{z=\sqrt{1-x^2-y^2}} (3x^2 + 3y^2 + 3z^2) \, dz \, dy \, dx$$

$$\approx \boxed{7.539}$$