5

Interest Rate Futures

As discussed in the introduction, interest rates have become quite volatile in the modern era. Changes may occur either because of policy action by the central bank or because of changes in the money market triggered by macro-economic changes in the domestic market, or even because of fluctuations in the foreign exchange market. Whatever the cause, institutions and individuals who borrow or lend significant sums, especially those who borrow or lend at floating interest rates, are exposed to risks arising from fluctuations in interest rates. Interest rate futures are an instrument allowing hedging and speculation in these risks. Interest rate futures are a sub-set of interest rate derivatives.

Interest rate futures or debt instrument futures

Strictly, interest rate futures are not futures contracts on interest rates *per se*, but rather futures contracts on *underlying interest-bearing debt instruments* like corporate, government and other bonds or short term deposits with a pre-specified face value and coupon (i.e., interest rate). When the maturity value of a bond or deposit is known, the implicit rate of interest (yield) can be calculated.

The underlying can be either a short term debt instrument (like a bank deposit) or a long term debt instrument (like a bond or debenture). Therefore, a futures contract on a debt instrument is *ipso facto* a futures contract on interest rates, but the relationship is inverse. This is different from stock or commodity futures: when one talks about these terms, one knows that one is substantially and directly dealing with the price movement in the stock and/or commodity itself, and not some implicit number therein.

The price in its own domestic market of a sovereign (i.e., central or federal) government security denominated in local currency varies exclusively and inversely on the basis of interest rates because credit and liquidity risks are nil. The coupon rate of interest on a bond reflects the rate of interest prevailing at the time the bond was initially issued, but interest rates change over time.

Example 5.1

In 2014, the prevailing market interest rate on long dated gilt-edged securities in a country is 10 per cent. The government issues a new 'gilt-edged' (i.e,. government-backed) security (known variously as gilts, government securities, government bonds or treasury bonds) with a coupon rate of 10 per cent. By 2016, the market interest rate for long-dated gilts is 14 per cent. New issues of gilts bear a coupon rate of 14 per cent. An investor in the gilt-edged securities market can now choose either to buy the new bond (producing a return of 14 per cent) or to buy the old bond. Naturally, he will not buy the old (10 per cent) bond at its issue price when he can get 14 percent on the latest issue. If however the price falls to a level such that the yield is 14 per cent, he will be willing to buy it. If this level is x, then

$$\frac{10}{x} = \frac{14}{100}$$

$$x = \frac{10 \times 100}{14}$$

= ₹71.42 per ₹100 of nominal or par value

For every ₹100 of nominal value, the holder gets interest of ₹10 (since this is a ten per cent coupon security). But since he only pays ₹71.42, his yield is

10/71.42 = 14 per cent

Thus, the price of a ₹100 bond will fall to ₹71.42.

Now assume that in 2018, the rate of interest falls to 9 per cent. Persons holding the old (10 per cent) bond, acquired at par, can get a higher return than on fresh issues. Sellers of the bond know that the market rate is 9 per cent and will thus be unwilling to sell the bond at any yield in excess of 9 per cent. The new price of the gilt will be y such that

$$\frac{10}{y} = \frac{9}{100}$$

y = (10 x 100)/ 9 = ₹111.11 per ₹100 nominal

The buyer of ₹100 nominal gets ₹10 as annual interest but has paid ₹111.11. His effective yield is thus

10/111.11 = 9 per cent

(The simple relationship illustrated here applies only to perpetual securities; for redeemable securities, the formula is more complicated but the principle is the same.)

Hence, the relationship between the price of a bond and the prevailing interest rate is inverse: when market interest rates rise, bond prices fall; when market interest rates fall, bond prices rise. These price changes occur so that the yield on an already-issued security is the same as that on a new one issued at the current rate. Therefore, bond futures are effectively an inverse form of interest rate futures.

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Interest rate futures are used for hedging by banks, financial institutions, pension funds and others whose assets or liabilities can be affected by changes in interest rates. In the interest rate futures markets, short hedgers are those seeking protection against rising interest rates while long hedgers are those seeking protection against falling interest rates. The method of quotation is structured so that a short hedger sells futures and benefits from a fall in price (rise in interest rate) in his futures transaction while a long hedger buys futures and benefits from a rise in price (fall in interest rates) in his futures transaction. This mirrors the situation in any other futures market.

The first interest rate futures were traded in October 1975 on the CBOT. Since then, the market has exploded. Another exchange that developed such derivatives along with the CBOT was the CME. CBOT specialises more towards the longer maturity (i.e., bonds maturing in the distant future) whereas the CME is more specialised in shorter maturity ones, such as Eurodollars (see below).

Interest rate futures are now actively traded in several developed country markets. American treasuries, Japanese bonds, German Euro-denominated sovereign bonds and other government notes have liquid derivatives' markets.

Another very active interest rate futures market is the market in Eurodollars. Eurodollars are typically short-term dollar debt (bank deposits) held outside the US. (The term 'Eurodollars' also has nothing to do with the Euro vs. dollar currency exchange rate; the Euro currency came into existence many years after the Eurodollar market.) The banks which hold such deposits could be foreign banks, or foreign branches of US banks. London dominates the Eurodollar market, and this market is generally based on the LIBOR or the London Inter-bank Offered Rate. It is the interest rate at which banks lend funds to other banks and much corporate debt is quoted on a 'LIBOR plus' basis, that is this rate plus a spread depending on the relevant party's credit and other risks.

The underlying for the most popular Eurodollar futures contract, and indeed the most popular futures' contract in the USA across categories is the $1 million, non-transferable three month Eurodollar futures contract traded on the CME. Such contracts trade are also traded actively in other countries, the Singapore Monetary Exchange being an example.

The Eurodollar contract, when it started trading in 1981, was the first derivatives' contract to use a notional basis and cash settlement instead of a 'delivery' settlement on expiry. For commodity derivatives, the underlying is

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a physical commodity but for financial derivatives, the underlying represents financial promises, often very liquid themselves. Hence the derivative contract on such promises can be settled in cash without causing any major spot-future discrepancy.

A characteristic feature of short-term interest rate futures is their manner of quotation. They are quoted by *deducting the yield per annum* from 100. The yield would be the one applicable to the contract term for that particular currency – for instance the rate applicable to the three month Eurodollar contract will be the three month US dollar LIBOR, while for a 30-day dollar contract, the one month US dollar LIBOR would apply.

Example 5.2

The Eurodollar futures contract for June is quoted at 96.00 while that for September is quoted at 96.50. This means the yield on a three-month deposit made in June is expected by the market to be

100 – 96.00 = 4.00 per cent per annum (interest received for three months would be 1 per cent)

The yield on a three-month deposit made in September is expected to be

100 – 96.50 = 3.50 per cent per annum (interest received for three months would be 0.875 per cent)

This also implies that the market is expecting interest rates to fall between June and September.

Interest Rate Futures (IRFs) in India

In India, interest rate futures were introduced in June 2003 but for a long time there was little liquidity in the market. These futures are traded on the NSE. The instruments listed are:

- 91-day (short term) treasury bills, and
- 10-year (long term) government bond or 'gilt'.

Treasury bills are zero-coupon short term securities, i.e., no interest payment is made but the bill is purchased at a price less than 100 and the difference between the purchase price and the maturity value of 100 is the implicit interest.

IRFs on NSE 'are standardised contracts based on six year, ten year and thirteen year Government of India Security' (NBF or NSE Bond Future II) and ninety one day Government of India Treasury Bill (91DTB)'.[1] These futures contracts on NSE are cash-settled.

1 https://nseindia.com/products/content/derivatives/irf/irf.htm.

The Reserve Bank of India is planning to introduce money market futures based on the overnight call money rate. This is partially because Interest Rate Futures (IRF) turnover is around 1 per cent of the equity derivatives volume and around 10 per cent of currency futures volume.[2]

Pre-2014 system: Notional basis

It was noted earlier in the discussion on exchange-traded futures that there may be more than one deliverable variety in a futures market and that if a variety other than the basis grade is delivered, price adjustments would be made. This is of particular relevance to interest rate futures. Until 2014, one 'lot' of the long term gilt security (abbreviated as 10YGS7, signifying Ten Year Government Security 7 per cent) was equivalent to a notional government bond of ₹ 2 lakh maturity value.

Participants may hold or need to hedge various government securities of different maturities and coupon interest rates, whereas the futures market will have only one of them as the basis variety. The basis variety in this case was, until 2014, notional and there may have been no security with exactly a 10-year maturity and exactly a 7 per cent coupon rate. These necessitated pricing adjustments which are called 'conversion factors'. Therefore, while futures trading took place on the basis of the notional variety, settlement of contracts is based on adjustments to take account of the actual securities involved.

In the case of the 10YGS7 contract, the conversion factor 'would be equal to the price of the deliverable security (per rupee of principal) on the first calendar day of the delivery month, to yield 7 per cent with semiannual compounding'.[3] As an illustration, Table 5.1 gives the conversion factors for certain government securities maturing in 2021 and 2022 as of September 2012.

Table 5.1: Deliverable basket and conversion factor for September 2012 contract

Sr. No.	ISIN nomenclature	Date of maturity	Conversion factor
1	IN0020110022 7.80% 2021	11-Apr-2021	1.0506
2	IN0020060318 7.94%2021	24-May-2021	1.0595

2 http://www.thehindubusinessline.com/money-and-banking/interest-rate-futures-lose-steam-in-2016/article8490493.ece.

3 NSE website.

Sr. No.	ISIN nomenclature	Date of maturity	Conversion factor
3	IN0020010040 10.25% 2021	30-May-2021	1.2056
4	IN0020110030 8.79% 2021	8-Nov-2021	1.1180
5	IN0020060037 8.20 % 2022	15-Feb-2022	1.0805
6	IN0020020072 8.35% 2022	14-May-2022	1.0925
8	IN0020039031 5.87%2022	28-Aug-2022	0.9210

Source: NSE website (exact data points not available as refreshes on a real time basis).

This table helped buyers and sellers to determine the amount to be paid for the actual security delivered against the contract. The quantity (principal amount) of bonds delivered would be the same as the principal amount of the futures contract: if one lot of 10YGS7 had been sold, the seller would have to deliver a government security with ₹ 2 lakh nominal value (since ₹ 2 lakh is the lot size of the contract) but the price paid would be based on the conversion factor. If a security has a conversion rate of 1.2, it means that one unit of the security is equivalent to 1.2 units of the notional security and hence the price payable will be 1.2 times the price of the notional security on the settlement date. For the futures contract maturing in September 2012, the notional futures contract was based on a 10 year notional government bond expiring in September 2022 with a 7 per cent semi annual coupon. The trade could actually be settled by giving equivalent quantity of April 2021 Government of India bonds having a 7.8 per cent coupon rate. Since a bond with higher coupon is being delivered the actual amount of cash received by the seller will be the settlement price of the futures contract multiplied by 1.0506 (see line 1 of the table). If the seller delivered the August 2022 Government of India bonds having a 5.87 per cent coupon rate, then the amount received by the seller would be the settlement price of the contract multiplied by 0.921 (see line 8 of the table), and so on.

Post-2014 system: Cash settlement based on a single security

One of the reasons why interest rate futures on the NSE had not been liquid is because they were settled by actual delivery (of only Government of India bonds, with the appropriate conversion factor). While various commodity hedgers and speculators can often get their hands on various standardised commodities to

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keep the market liquid, it is more difficult in the case of Indian government bonds because most of them are owned by public sector banks. In 2013, the Reserve Bank made a fresh attempt to popularise them, and in January 2014 a new instrument was launched. From 2014 onwards, contracts are based on an identified actual Government of India security with approximately ten years to maturity (rather than a notional security) with a contract size of ₹ 2 lakhs and settled only by cash with no actual deliveries. The long term bond futures market has become more liquid. The market does not allow delivery, so conversion factors etc. are no longer applicable. (However, the US Treasury bond market uses a notional basis with conversion factors.)

A second reason for interest rate futures not becoming active in India is that the bond market itself is inactive. A third factor is that in India short term and long term interest rates tend to move together, reducing the need for hedging of mismatches between assets and liabilities. Analysis by Shah showed that from 1997 to 2009, the simple correlation coefficient of the 90 day rate and the ten year rate was around 0.75 in the USA but around 0.95 in India.[4]

Understanding the underlying

Interest rate futures have developed their own conventions and terminology over time. Given the relative recency and limited liquidity of the Indian IRFs, examples from the US interest rate futures markets will primarily be used to illustrate certain concepts relating to interest rate futures.

Duration

The duration of a bond (or a bond portfolio) does not literally mean the remaining time or average time to maturity, as it may seem from the colloquial meaning of the word (though it is a fact that, for a given coupon and other factors being equal, securities with a longer period left to maturity have a longer 'duration'). More precisely, duration is a measurement of how long, in years, it takes for the price of a bond to be repaid by the cash flows (interest and principal) coming from the bond. For a normal or 'vanilla' bond (where there are periodic

4 'How useful are the new interest rate futures?' Available at: http://ajayshahblog.blogspot.sg/2009/09/how-useful-are-new-interest-rate.html, 7 September 2009.

interest payments followed by a final repayment of principal), the duration will be less than the time remaining for maturity—this is because a part of the price paid for the bond is being received each year. For a zero-coupon bond, i.e., one issued at a price lower than the amount payable at maturity but with no periodic interest payment, the duration is equal to its time to maturity. For this reason, bond duration (also called the 'Macaulay duration') is expressed in time or number of years.

Duration is a measure of the *sensitivity of the price of a security to a change in interest rates* – the higher the duration, the more the price of the security will change for a given change in interest rates; a bond with a duration of seven years will fluctuate more when interest rates change than a bond with a duration of two years. When using duration for the purpose of measuring sensitivity, another definition, known as 'modified duration', is often used. Modified duration is discussed later in this chapter.

Day count fraction conventions

Bond coupons are payable at specified intervals of time, say every six months. But a calendar year has an odd number of days. Similarly, different months have different numbers of days – 28 or 29 in February, 30 in June, 31 in March etc. The question that arises is how to reckon the amount of interest due for fractional periods. The contract could be based on assuming that a 'year' consists of 360 days with each month having 30 days. Alternatively, it could be measured by the exact number of days. It is important that parties to a transaction know the convention in advance. To avoid ambiguity about the amount of interest and timing of payment, conventions have evolved for counting the time period, known as 'day count fraction conventions' or simply 'day count conventions'. (This is sometimes abbreviated to 'DCF', not to be confused with Discounted Cash Flow.) The convention used for a particular contract will determine the exact period of time to which the interest rate applies, and the period of time used to calculate accrued interest (when the instrument is bought or sold between coupon dates).

In the US Treasury bond market – the world's most liquid market – the interest due is calculated by using actual numbers of days for both months and years. On the other hand, in the corporate and municipal bond markets, durations are rounded up to 30 days for a month and 360 days for a year. For

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example, under the latter convention there are 30 days in February 2013, but only 28 days by the former convention and thus the amount payable as coupon in February will vary between the two markets. The US money market (short-term private debt) market has a mixed convention known as Actual/360. Under this convention, the numerator is always the actual number of days like in the treasury market, but the denominator is based on a 360-day convention. For this reason, under this convention, a 10 per cent bond will earn slightly more than 10 per cent in a single year – since the interest payable will be 365/360 x 10 per cent. These three conventions are referred to simply as Actual/Actual (use actual number of days divided by 365 or 366 in a leap year), 30/360 and Actual/360, and there are more variants. The LIBOR (discussed earlier) is also quoted on an Actual/360 basis, except for GBP or British Pound which is quoted on an Actual/365 basis.

India generally follows the actual (i.e., actual number of days in each month and 365/366 days per year) basis. For example, a bond of ₹ 1,000 maturing in 180 days paying an annual coupon of 7 per cent will accrue/deliver:

(180/365) x (0.70 x 1000) = ₹ 34.52

If a 360 day convention had been used, it would have accrued/delivered ₹ 35:

(180/360) x (0.70 x 1000) = ₹ 35.

Short-term instruments are often traded not by quoting their market prices, but by their 'discount rates'. For example, if the face value is $1000 and the 'price' of a 182-day Treasury bill is 10, this implies that the *annualised* rate of interest is 10 per cent of the face value of 1000 USD, i.e., $100. For Treasury bonds and notes, the US still follows the anachronistic system of reporting prices in thirty-secondths i.e., fractions with thirty-two as denominator.

Accrued interest

When bonds are bought or sold between the dates of two interest payments, this needs to be properly reflected in the pricing to reflect the interest which has accrued on the date of purchase. For treasury securities in America, the formula used is:

Cash price or 'dirty price' = quoted price or 'clean price' + accrued interest amount since last payment date.

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Conversion factor and cheapest-to-deliver (CTD) concept

As in the case of the pre-2014 long term bond futures market in India, the US futures markets are usually based on a notional underlying allowing actual delivery of bonds of the same basic type (i.e., treasury bond for treasury bond, corporate bond for corporate bond etc.) with somewhat different expiry dates and coupon rates. (Under the rules of the futures contract, very different coupon rates and expiry dates may not be acceptable because they may not serve the needs of the long position holder.) When a short position holder delivers a bond against the futures contract, he receives:

Most recent settlement price × Conversion factor + Accrued interest

The conversion factors for delivery are set in advance and actual movements in the cash market for various bonds may not exactly match the prices given by these conversion formulae. This means that those who are 'short' in the futures contract (i.e., have to deliver the bond at the time of expiry) should be watchful to *deliver the one that costs the least to them*. Example 5.3 illustrates this. (This concept is not applicable to IRFs on India's NSE since 2014 since the contract does not allow actual delivery and allows only cash settlement.)

Example 5.3

A fund holding a large and diversified portfolio of bonds was short US treasury futures because it expected interest rates to rise (when interest rates rise, generally bonds fall in value – other things being equal). Instead of squaring up its position, the fund decided to hold its position until expiry. Now, it must deliver some bonds to the exchange. The most recent settlement price is 95-16 or 95.50 (since the price is quoted in 1/32nds, 95-16 means 95 and 16/32 i.e., 95.50). Four bonds are eligible for use for settlement in lieu of the notional underlying:

Bond 1: Quoted price of 101.25 and conversion factor of 1.047

Bond 2: Quoted price of 153.75 and conversion factor of 1.602

Bond 3: Quoted price of 159.50 and conversion factor of 1.654

Bond 4: Quoted price of 125.00 and conversion factor of 1.301

Taking Bond 1, its market value (based on the quoted price) is 101.25. If it is delivered against the contract, the amount received will be the settlement price adjusted for the conversion factor, i.e.

95.5 x 1.047 =99.9885.

This is less than the market value of the bond. By delivering this bond, the fund will incur a loss of 101.25 – 99.9885 = 1.2615 per bond due to the pricing difference between the bond (cash) market and the formula used in the futures market (this is a kind of 'basis' issue). Similarly, the market value and delivery value of the other bonds can be computed, producing the following list of the net loss (or cost) from delivering different bonds:

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1. *101.25-(95.5 x 1.047) = $ 1.262*
2. *153.75-(95.5 x 1.602)= $ 0.759*
3. *159.50-(95.5 x 1.654)= $ 1.543*
4. *125.00-(95.5 x 1.301)= $ 0.755*

Therefore, the cheapest-to-deliver or 'CTD' bond in this case is Bond 4.

Pricing of interest rate futures

The same relationship used to determine the futures or forward price in other markets can be used for IRFs too. The standard futures or forward pricing formula as shown in chapter 4 was:

$F \leq S + C - Y$

In the case of debt instruments, there are no storage costs and no convenience yields. However, there is a carrying cost reflecting the opportunity cost of the funds deployed and a real yield reflecting the interest rate on the bond involved. The carrying cost and the yield can be simply calculated by multiplying the relevant interest rate for the relevant period of time by the amount involved (i.e., the spot price). Financial instruments are not subject to discontinuities in production or storage (see Table 4.1). Therefore, they generally reflect the carrying cost approach to futures pricing and the formula becomes an equation rather than an inequality:

$$F = S + C - Y$$

i.e., $F = S + (C - Y)$

i.e., $F = S + S(c_t - y_t)$

i.e., $F = S\,(1 + c_t - y_t)$

Where S = spot price of the bond

c = interest rate on funds borrowed (opportunity cost) for the time t (not per annum)[5]

y = interest rate on the bond for the time t (not per annum)[6]

t = time period involved

5 If the applicable rate of interest is 12 per cent per annum and the period under consideration is three months, then for this purpose i_t will be 3 per cent.

6 If the applicable rate of interest is 6 per cent per annum and the period under consideration is three months, then for this purpose y_t will be 1.5 per cent.

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Typically, the yield on bonds exceeds the short term opportunity cost of capital so

$y_t > c_t$ and $Y > C$.

Let the net amount of the yield on the bond (after subtracting cost of funds) be

$Y - C = I$

and let $y_t - c_t = i_t$.

Then, substituting I for C–Y and i_t for c_t–y_t, the formula becomes

$$F = S - I$$

i.e., $F = S(1 - i_t)$

where,

F= futures price

S= spot price of the bond

I= amount of interest (absolute amount) earned for the period after deducting cost of funds

$i_t = y_t - c_t$ = percentage yield on the bond nett of cost of funds for the time period t (not per annum).[7]

This formula ignores the compounding factor. If continuous compounding is used it can be shown mathematically that the formula becomes:

$$F = (S - I)e^{rt}$$

Where,

$r = c$ = risk free rate of interest *per annum* (generally taken as the Treasury bill rate)

t = time period measured in number of years involved (e.g., three months =0.25)

e is a mathematical constant (like π) known as Euler's number or the exponential constant (approximately equal to 2.7182818282).

The risk free interest rate means the interest rate which does not reflect any premium for the possible credit risk (i.e., risk of the borrower not repaying). Hence, the rate of interest paid by the government on its Treasury Bills is generally taken as the risk-free rate.

7 If the cost of borrowing is 12 per cent per annum and the yield on the bond is 6 per cent per annum and the period under consideration is three months, then i_t will be 1.5 per cent (i.e. 12 per cent minus 6 per cent but for a period of three months or 0.25 years).

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Example 5.4 Bond futures

In March, a company finds it has a temporary surplus cash of ₹ 10 lakhs, which it will require for other uses by September. It wishes to invest the amount in gilt-edged stock because of the attractive interest yield but is apprehensive about a possible rise in market interest rates which may reduce the bond price. It therefore buys ₹10 lakhs nominal of 9 per cent Government of India bonds, whose current spot market rate is ₹ 92 per ₹100 of nominal bonds. (This happens to be the bond used as the basis for the futures contract too.) It hedges the transaction on the futures market and the results are as follows:

Date	***Action***	
	Spot market	***Futures market***
1 March	*Buy ₹ 10 lakhs nominal at 92 per cent Pay ₹920,000*	*Sell ₹10 lakhs nominal of September futures at 93 per cent. Pay ₹93,000 as margin (deposit)*
31 August	*Sell ₹10 lakhs nominal at 89 per cent Receive ₹890,000.*	*Buy ₹10 lakhs nominal of September futures at 89 per cent Receive ₹(930,000 – 890,000) = ₹40,000 as profit plus ₹93,000 margin refund.*

As shown above, interest rates did indeed increase by August, resulting in a fall in gilt prices from 92 to 89. The financial institution has received coupon interest for the half year of ₹45,000, but if it had not hedged itself, it would have lost ₹30,000 through fall in bond values. However, because it had hedged itself, its spot market loss of ₹30,000 is compensated for by a gain of ₹40,000 on the futures trade. (The slight excess gain reflects the change in futures-spot spread.)

Eurodollar futures

As mentioned, one of the most heavily traded IRF contract in the world is the three-month or ninety-day Eurodollars futures contract, traded on the CME. The standardised notional underlying is 1 million US Dollars. Deliveries are available for March, June, September and December,for many years into the future.

A one-basis point move in the relevant Eurodollar interest rate means a move (up or down) in the interest rate of 0.01 percentage points. On 1 million dollars, this corresponds to 1/10,000th multiplied by 1 million or 100 dollars.

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Since the contract is of three months (or one-fourth of a year), each basis point increase in the interest rate causes a loss of $ 25 per long Eurodollar contract (and one basis point decrease creates a profit of the same absolute amount).

Generally, the Eurodollar interest rates are very close to the LIBOR as both represent average rates at which prominent banks loan each other funds.

Example 5.5: Short term interest rate (long hedge)

On 15 April, an Indian IT company clinches a lucrative software contract and is expecting a large receipt in October for about $10 million and would need to invest it for a three month period from then. It is apprehensive that the interest rate on deposits may fall between April and October and hedges by 'locking in' a rate of interest for the October to January period. The October Eurodollar futures are quoting at 97.500. Based on the method of price quotation of Eurodollars, this means that the rate of interest implied is

100–97.5 = 2.5 per cent.

The investor hedges by buying 10 contracts, the lot size being $ 1million.

On October 15th the three-month Eurodollar rate is 2.1 per cent. The final settlement in the contract is thus at a price of 97.90. In the Eurodollar contract, the prices are quoted in terms of annual interest rate but the contract is for a three month deposit. Therefore a movement of 0.01 per cent or in interest rates or 1 basis point (e.g., from 97.99 to 97.98) implies a change (per contract) of

0.01per cent x ¼ x 1,000,000 = $25

The Eurodollar future has appreciated from 97.5 to 97.9, i.e., by 40 basis points or 0.40 per cent. As the company has bought 10 contracts (worth $10 million), the resulting gain is:

40 basis points x $25 per basis point x 10 contracts = $10,000

It now invests the sum of $10 million for 3 months at the ruling interest rate of 2.1 per cent per annum for three months and earns:

$10,000,000 x 2.1 per cent x 3/12 = $52,500.

The total return, adding the gain on the futures contract is:

$52,500 + 10,000 = $62,500.

This return amounts to:

62,500/10,000,000 = 0.625 per cent for three months,

i.e., 0.625 per cent x 4 = 2.5 per cent for a full year.

Thus, the company was able to successfully fix the return at the 2.5 per cent level prevailing when the hedge was undertaken, even though interest rates fell later to 2.1 per cent.

The transactions are summarised in the following table:

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Date	Action	
	Spot market	***Futures market***
15 April	*Acquire 'deferred asset' (i.e. cash due in future) of $10m*	*Buy 10 contracts of October Eurodollar futures worth $10m @ 97.5* *Pay margin of $10m x 97.5 per cent x 10 per cent =$97,500*
15 October	*Receive cash and deposit it at 2.1 per cent per annum for three months*	*Sell 10 contracts of October Eurodollar futures worth $10m @97.9.* *Receive refund of $97,500 plus profit of (97.9–97.5) per cent x 3/12 x $10m= $(97,500+ 10,000) = $107,500.*
15 January	*Receive back $10m.* *Receive interest thereon of 2.1 per cent x 3/12 x $10m= $52,500.*	

Example 5.6: Short-term interest rate (short hedge)

In July, an Indian company secures an order which will require borrowing of ₹50 crores for working capital for three months from September. The company's short term borrowing rate is 1 per cent above the Treasury Bill rate. It is apprehensive that interest rates will rise before then and hedges the risk in the 91 day T Bill market (assumed to be active). Interest rates (Treasury Bill yields) turn out to be as follows:

July 1	*8.50 per cent*
September 1	*8.25 per cent*

Date	Action	
	Spot market	***Futures market***
1 July	*Acquire 'deferred liability' (i.e., payment due in future) of ₹50 cr.*	*Sell 2,500 contracts of 91 day T Bills at 91.50* *Pay margin of 91.5 per cent x ₹50 cr. x 10 per cent = ₹457.5 lakhs*

Date	***Action***	
	Spot market	***Futures market***
1 September	*Borrow ₹50 cr. @ T-Bill rate plus 1 per cent i.e., 9.25 per cent*	*Buy back 2,500 contracts of 91 day T Bill at 91.75* *Receive margin refund of ₹ 457.5 lakhs less loss of* *(91.75–91.50) per cent x 3/12 x* *₹ 50 cr.=* *₹ (457.5–3.125) lakhs. = ₹ 454.375 lakhs*
1 December	*Repay ₹50 cr. along with interest thereon of 9.25 per cent x 3/12 x* *₹ 50 cr.=* *₹ 115.625 lakhs*	

Total cost of borrowing:	*₹ lakhs*
Interest paid:	*115.625*
Loss on futures:	*3.125*
	118.750

Effective rate of interest for 3 months = 118.75/ 5000 × 100 = 2.375 per cent

Annualised rate of interest $= 2.375 \times \frac{12}{3} = 9.5$ *per cent*

The company ends up paying 9.5per cent (i.e., 8.5per cent + 1per cent margin over T Bills)–the rate which it wanted to ensure. (Its expectation that rates of interest would rise was belied and it would have been better off by not hedging, in this particular instance,but that is with hindsight!)

Adjusting hedges to fit futures contract specifications

As pointed out above, interest rate futures contracts are often either notional (against which many securities can be delivered) or can only be settled in cash against a specified bond or instrument. In such cases, a hedger seeking to hedge a particular asset or liability has to adjust his hedge carefully so that the outcomes on the futures contract match those on the spot market. Even where the underlying is an exact match, there can be slight mismatches in other parameters: example, a month may have 28 days or 31 days whereas the contract may be for 30 days. (The examples above deliberately circumvented

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these issues by choosing appropriate instruments, dates and hedge sizes.) A complete discussion of all the techniques involved in ensuring near perfect hedging outcomes is beyond the scope of this book as it involves a number of complexities. In this section, an outline is given of the main points to be borne in mind in real-life hedging situations.

Hedge period vs. market months

Often a hedge may be required for a period beyond the last available maturity. Alternatively, the last maturity may be illiquid and hence the hedge may be placed in a nearer month. In such cases a roll-over transaction may be required, which usually involves a transaction cost and may also involve a small basis risk (i.e., risk of change in spread) as a result of roll-over.

Size of exposure vs market lot

The exposure to be hedged often differs from a whole number of market lots. For instance, a company may want to hedge an exposure of \$ 2.3 million in Eurodollar futures. The market lot is \$1 million. The hedger will have to choose the nearest whole number of contracts–in this case two. However, this leaves some exposure unhedged (or creates a new exposure in case the number is rounded upwards). The larger the size of the risk to be hedged, the easier it is to get an almost exact correspondence of the hedge quantity. If a \$ 20.3 million exposure is hedged using 20 futures contracts with aggregate value of \$ 20 million, the unhedged residue is just 1.5 per cent of the exposure whereas it was 13 per cent in the \$ 2 million case.

Equivalency or hedge ratio

Because of differences between the instrument being hedged and the basis grade of a financial futures contract, weighted hedges may have to be used in many cases. As discussed in chapter 4, the cash-futures equivalency ratio (also called the hedge ratio) is a parameter which reflects the nominal value of futures contracts needed to hedge a given nominal value of spot market exposure. In general, for any financial futures transaction, the number of futures contracts to be bought/sold can be calculated as follows:

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$$\frac{\text{Nominal value to be hedged x equivalency ratio}}{\text{Nominal value of one futures contract (market lot)}}$$

Example 5.7

Hedger wants to hedge £376,000 in the long gilt market. The contract size on the London International Financial Futures Exchange for long gilts is £50,000. The equivalency ratio has already been determined as 1.3. How many futures contracts should he trade?

Solution: (376,000 x 1.3) / 50,000= 9.776

Since partial contracts cannot be traded, the hedger should trade 10 contracts in this case.

In the example, an equivalency ratio was already given. In practice, determining it for interest rate futures is quite complicated, much more so than for other types of financial futures. Some of the main factors affecting this are given below.

Term of the hedge

When hedging a financial instrument which has a term different from the futures contract, it is necessary to weight the hedge pro rata. For instance, if a 30-day contract is being used to hedge an exposure over the month of December which has 31 days, the equivalency ratio will be 31/30, other things being equal.

Example 5.8

A hedger wants to hedge \$ 5 million in a 90-day Eurodollar contract having a contract size of \$ 100,000, to take care of an exposure from June 1 to August 31: How many contracts should he trade?

Solution:

The equivalency ratio has to be determined first. The period runs for 92 days as against the contract period of 90 days. The ratio is therefore 92/90.

No. of contracts needed = \$(5,000,000/100,000) x (92/90) = 51.11

So he has to trade 51 contracts not 50.

Conversion factor

As already discussed, where the basis (i.e., deliverable) grade for a contract is notional, conversion factors are supplied by the exchange (equivalent to tendering differences in a commodity exchange) if settlement by delivery is

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allowed. Because of differences between the instrument being hedged and the basis grade of the futures contract, weighted hedges may have to be used in many cases. For instance, if the conversion factor for a particular gilt is 0.75, then this should be taken as the equivalency factor for calculating the number of contracts to be traded, so that the size of the futures trade matches the size of the spot market risk. Even in a cash-settled market, when hedging a security in the same class but with different maturity and coupon, the hedger must be careful to weight his hedge by a suitable conversion factor.

Price sensitivity

In the previous paragraph, the need to weight the hedge by the conversion factor was referred to. The use of conversion factors ensures that the overall size of the hedge is appropriate. Though this is better than an unweighted hedge, it is still not sufficient to ensure close correspondence between spot and futures price risks, because of the sensitivity factor. The extent to which the price of a long-term security changes for a given change in interest rate, is affected by several factors. Among these factors are the maturity period (or tenor) of the security as well as the coupon rate. (Even intuitively, it is easy to see that a 1 per cent change in interest rate effective for a period of one year is less significant than a 1 per cent change which will be effective for a period of 20 years.) Without getting into the complexities of bond pricing, one can say that:

a. Other things being equal, the longer the maturity, the greater the sensitivity of the price for a given change in interest rate (e.g., a 30-year 9 per cent bond is more sensitive than a 10-year 9 per cent bond).
b. Other things being equal, the smaller the coupon, the greater the sensitivity of the price for a given change in interest rate (e.g., a 2 per cent 10-year bond is more sensitive than a 9 per cent 10-year bond).
c. Other things being equal, the smaller the yield[8] the greater the sensitivity of the price for a given change in interest (a bond yielding 5 per cent will be more sensitive than a bond yielding 7 per cent).

Because of this, when hedging through the gilt or bond futures markets, it is necessary to adjust for the differences in price sensitivity between:

8 Yield is based on the current market price whereas coupon is based on the issue or nominal price.

- the asset or liability to be hedged; and
- the notional basis instrument in the futures market (e.g. the 6 per cent coupon 15-20 year T Bond in CME) or the actual bond which forms the basis grade (as in the case of the 10 year Government of India securities futures).

Otherwise, the price change in the spot market will be under- or over-compensated in the futures market. The adjustment is done by means of adjustments for duration (or basis point value) and convexity (all of which are specific, mathematically defined, terms).

Duration has already been discussed earlier. Duration is a number reflecting the weighted average maturity period in years of a bond after adjusting for coupon rates, frequency of interest and principal payment etc., all of which affect the proportionate change in price for a given change in yield. While duration (also known as Macaulay duration) is measured in years and is an indirect measure of sensitivity, 'modified duration' is a direct measure of price sensitivity to interest rates. It is simply: Percentage change in bond price/ Percentage change in interest rates.

Modified duration is in many contexts simply referred to as 'duration', and this can be confusing. Readers should note that if the term 'duration' is used, they should check the context and the unit of measurement (years vs. a ratio) to understand which concept of duration is being referred to. However, the duration and the modified duration are usually close to each other numerically, and in a very approximate and rough sense, the two terms can be used interchangeably; thus a bond with a sensitivity (modified duration) of seven (i.e., .07 per cent price change per .01 per cent change in interest rates) can be taken as having a duration of approximately seven years.

While the modified duration gives the proportionate change in price for a given change in interest rates, another related term known as the basis point value is the absolute change in price (in £, $ or ₹) per basis point change in yield and is similar to the duration except that it is calculated as an absolute amount. It is absolute value of change in bond price per 0.01 per cent change in interest rates (i.e., per basis point).

It should be noted that the modified duration and basis point value for the same bond are *different at different yield levels*. Thus, the price change caused by a drop in yield from 10 per cent to 9 per cent will be different from the price change caused by a drop in yield from 5 per cent to 4 per cent.

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Example 5.9

The UK has some 'perpetual' government securities which are not redeemable and just continue to pay interest indefinitely. One such is the 3.5 per cent War Loan. This security is simple to price as there is no maturity value or maturity date and hence provides a simplified illustration of how the modified duration changes at different levels of interest rate.

Assume the current interest rate is 10 per cent. For the War Loan to yield 10 per cent, its price must be

3.5/.1 = 35

Now assume the interest rate changes to 9 per cent. The price will change to

3.5/0.09 = 38.89

A 1 per cent change in interest rates produced a change in price of

(38.89–35)/35 = 11.11 per cent, i.e., modified duration = 11.11

When the interest rate is 5 per cent, the price of the War Loan will be:

3.5/0.05 = 70

If it changes from 5 per cent to 4 per cent, the price will become:

3.5/0.04 = 87.5

The change is:

(87.5–75)/75 = 16.66 per cent, i.e., modified duration = 16.67.

So the modified duration has increased.

The convexity is a measure of *how the duration/basis point value itself changes at different prices*. Using the three concepts together it is possible to work out how a given price change will affect a particular bond. However, for small changes in interest rates, the convexity can be ignored for practical purposes and calculations can be done using just the duration.

Another aspect of price sensitivity is the correlation between different kinds of interest rates. At any given time, interest rates for different periods of time (short term to long term) may be different. This relationship when expressed graphically is called the 'yield curve'. Normally, long term rates are higher than short term rates. When interest rates change, they may sometimes display a similar change in all interest rates; in that case the yield curve shifts upwards or downwards in a parallel movement. However, it is also possible that the yield curve may change in shape. For example, a 1 per cent change in short term interest rates may be accompanied by only a 0.2 per cent change in the 10 year interest rate, and the change for maturities below 10 years may lie somewhere between the two. Thus, this aspect (changes to the yield curve) will also have to be factored in. Similarly, when using futures as a cross hedge – e.g., using gilt-edge government security futures to hedge a holding of corporate bonds –

the correlation may be partial. A 1 per cent change in the 10 year government bond yield may produce, say, a 1.1 per cent change in corporate bond yields. Unlike the duration, basis point value and convexity, correlations between one instrument and another cannot be determined by mathematical calculation. One can only make predictions based on past behaviour and statistical analysis which of course are subject to error. Such cross hedges are therefore not always effective.

The manner of making these adjustments is beyond the scope of this book. In crude or approximate terms however, it is possible to calculate an appropriate hedge ratio without considering convexity, yield curve changes or correlations between different interest rates.

Simplified calculation of the hedge ratio

Take the cases of an investor with a large fixed interest bond portfolio. Rising interest rates would reduce the value of the portfolio (because bond prices will fall). The risk can be hedged by 'shorting' (i.e., selling) bond futures. (A person who has taken on a floating-rate mortgage loan will also lose when interest rates rise and can, similarly, hedge by shorting bond futures.) Since some contracts for the distant future may not have enough liquidity, a good hedge may involve shorting the most liquid and easily available interest rate futures, adjusting for the effective duration of the bond portfolio.

Assume that interest rate changes equally for different maturities, i.e., the shape of the yield curve is unchanged. The long bond portfolio will change in value and the extent of the change will be approximately:

Portfolio value x portfolio modified duration x change in yield (interest rate)

$= V_p \times D_p \times \Delta Y$.....(1)

Where,

V_p = value of the portfolio (strictly speaking, this should be the value at the maturity of the hedge, but as an approximation the current value can be taken)

D_p = modified duration of the portfolio at the maturity of the hedge

ΔY= change in yield

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The change in the value of the futures contract used as hedge will be approximately (per contract):

Contract price x modified duration of futures contract x change in yield

$= V_f \times D_f \times \Delta Y$ …… (2)

Where,

V_f = value (contract price) of the futures contract

D_f = modified duration of the futures contract at the maturity of the contract

ΔY = change in yield

Here, D_f is the duration of the underlying assets underlying this contract at the maturity of this contract.

The number of contracts N needed to carry out the hedge would hedge would be:

$$N = \frac{V_p \times D_p \times \Delta Y}{V_f \times D_f \times \Delta Y}$$

Cancelling ΔY,

$$N = \frac{V_p \times D_p}{V_f \times D_f}$$

This ratio is known as the duration-based hedge ratio or the price sensitivity hedge ratio. It may have to be rounded up or down to get the actual number of futures contracts traded.

Example 5.10

A fixed income portfolio manager has been successful over the first nine months of the financial year, and she wants to lock in her profits. It is 1 January and the financial year for her firm ends on 31 March. She expects volatility and an increase in interest rates in the coming months. She controls a $100 million portfolio invested in non-US sovereign bonds. Three months from now, her portfolio duration will be five years.

She is considering using the March T-bond futures contract to hedge the portfolio. The current price of the March contract is 97-01(i.e., 97 and 1/32) or 97.03125. Since the face value of the futures contract is $100,000, the contract price is $97,031.25. The yield on this is 8 per cent, and the duration will be around 10 years at the end of March, i.e., after three months. How should she hedge the portfolio using T-bond futures?

Solution:

She has a long position in bonds and therefore needs to hedge by going short in the treasury *futures. The duration is five years, which can be assumed to be a modified duration of five also.*

The number of contracts that she should sell should be:

$$N = \frac{V_p \times D_p}{V_f \times D_f}$$

Or,

$$N = \frac{100 \text{ million} \times 5}{97031.25 \times 10} = 515.3$$

Therefore, she should sell 515 contracts. In this way, her interest rate risk is likely to be almost fully immunised. Note however that:

- *if the interest rate on non-US sovereign bonds changes at a different rate from the interest rate on US treasury bonds, or*
- *if the five year interest rate changes by a different amount from the 10 year interest rate,*

then her hedge may not neutralise her risk and she may have a net profit or loss resulting from the hedge.

Asset-liability management (ALM) by financial institutions like banks

Banks generally lend for long durations, but take deposits for much shorter durations. That means the average duration of their liabilities is smaller than that of their assets. A sudden and sharp rise in interest rates would reduce the value of both assets and liabilities but the value of a bank's liabilities would fall less steeply than the value of assets. This would diminish the bank's equity. Hence, banks generally need to hedge against interest rate changes, and more specifically interest rate increases. (No doubt, if they hedge against rising interest rates, they are effectively forgoing the benefit from falling interest rates.)

This can be done through various ways – using interest rate futures or forwards to hedge generally involves a strategy known as duration matching. Such hedging of an entire portfolio is often called macro-hedging (as against the hedging of single bond which is micro-hedging). Investment funds and treasury departments of companies also use the process, and often call it 'portfolio immunisation'.

An important thing to remember is that:

- no macro-hedging strategy for a portfolio as a whole with many different instruments; and
- no micro-hedging strategy involving cross-hedges, i.e., hedging an instrument very different from the basis grade, is *perfect or risk-free*.

'Duration matching' only protects against parallel shifts in interest rates, not against non-parallel shifts which alter the shape of the yield curve. Changes in the shape of the yield curve may be anticipated using statistical correlations but these may turn out to be inaccurate.