11

Options – II

Pricing of Options

As briefly mentioned in chapter 10, the price of an option is the sum of two components: intrinsic value and time value.

Intrinsic value

To recapitulate, the *intrinsic value* of an American option is:

- For call options: The difference, if positive, between the current price of the underlying and the strike price of the option.
- For put options: The difference, if positive, between the strike price of the option and the current price of the underlying.

The intrinsic value of a European option is the difference, if positive, between the current price of the underlying and the *discounted present value* of the strike price for calls and vice versa for puts.

As was seen in chapter 10, an option which has intrinsic value is said to be 'in-the-money' and one with no intrinsic value is either 'at-the-money' or 'out-of-the-money'. In the case of traded options the premium is usually referred to as the 'price' of the option and hence the term pricing refers to how the premium is determined.

Time value

As regards the second component, *time value*, this is due to the fact that even if the price may be unattractive today, future fluctuations may make the option profitable. The time value depends on the interplay of a number of factors:

Time

It is obvious that the longer the period of time, the greater are the chances of price fluctuations and vice-versa. Thus, time value generally varies directly with

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amount of time left to maturity. This leads to the fact that options are wasting assets: as an option's expiration date approaches, its time value diminishes and eventually becomes nil. The only value remaining is the intrinsic worth, if any. Therefore, if an option is not sold or exercised by the expiration date, it becomes worthless. (This is important from an investor's point of view: in contrast to options, underlying assets like shares can be held indefinitely.)

Extent of the difference between current price and strike price

An option which is deeply out-of-the-money has a lower time value than an option which is only slightly out-of-the-money. A deep OTM option has a much lower probability of ever becoming profitable than one which is slightly OTM for the same maturity period.

Example 11.1

The current market price of shares in X Ltd. is ₹ 300. The 200 put for August is priced at ₹ 5 while the 250 put is priced at ₹ 14 and the ₹ 290 put is priced at ₹ 30. Note that none of them has intrinsic value because in all cases, the option is not worth exercising at the current market price but the deep OTM option (200) is worth much less than the near-the-money (290) option.

Interest rate

Generally, increases in interest rates tend to cause higher premia for calls and lower premia for puts, while decreases in interest rates tend to cause lower premia for calls and higher premia for puts.[1]

This is because a call option represents a contingent payment of money for the holder while a put option represents a contingent receipt of money. Higher interest rates reduce the present value of future cash flows: for a call option holder, this reduces the future payment making the option more valuable, while for a put option holder this reduces the future receipt making it less valuable.

1 In some rare cases (when interest rates are relatively high compared to price volatility), the interest rate factor (which reduces the value of longer maturity receipts) may overwhelm the 'longer profit opportunity' factor, thereby resulting in prices of long duration put options being lower than those of shorter duration put options.

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Volatility (price variability) of the underlying

If the commodity (or share or financial instrument) normally has a very stable price, then the time value would be relatively smaller. If it has a high degree of volatility, then the time value will be relatively greater because there is a greater likelihood that the price will move to ITM.

Dividends

Pricing of options on dividend-yielding securities is more complex than pricing of options on commodities. In the case of a share, the expected dividend has to be factored into the calculation of the time value. As a general rule, the higher the dividends expected, the lower the value of call options (compared to a similar situation with no dividends), and the higher the value of put options. This is because, *ceteris paribus*, the price of the stock is expected to fall by the amount of dividend pay-out on the ex-dividend date (adjusted for time value of money).

Here it is important to note that only expected dividends reduce the value of calls *vis-à-vis* a situation where no dividend is paid. Unexpected dividends could increase the price of the stock and hence the call. After the unexpected dividend is declared and the new stock price level is established, the Black-Scholes value (see later in this chapter) of the call would again be relatively cheaper than a hypothetical situation of no dividend.

Absolute value of underlying

Lastly, the absolute amount of time value also varies with the absolute value of the underlying. In absolute terms, a given percentage swing can result in differing amounts of losses for the writer depending on the price of the underlying. The absolute value of loss to the writer of a naked (i.e., uncovered) call option from a 10 per cent rise in the price of a ₹ 40 share is clearly less than that from a 10 per cent rise in a ₹ 400 share. Hence, time value of an option on a ₹ 40 share will obviously be lower than on a ₹ 400 share.

Summary

a. The premium for an option is the intrinsic value plus the time value.

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b. Intrinsic value is nil for OTM/ATM options.

c. The intrinsic value of an ITM American option can be calculated in a straightforward manner: strike price minus market price of underlying for a put and vice versa for calls. (For ITM European options, precision requires that the strike price be discounted to present value, but for short term options and at low interest rates, the discounting can be ignored for most purposes and the intrinsic value can be taken as roughly the same as that of an American option.)

d. Calculating the time value is more complicated. All unexpired options (whether OTM/ATM or ITM) have time value. Time value is affected by:

   i. the time remaining till expiry;
   ii. the extent of the difference between strike price and market price;
   iii. the prevailing rate of interest;
   iv. the volatility (i.e., variability of the price) of the underlying;
   v. the amount of dividend if any; and
   vi. the absolute value of the underlying.

Put-call parity (PCP)

It has been shown[2] that in markets where short sales of the underlying asset are possible, the premium for a call and a put with the same maturity date and an at-the-money strike price will be equal because of arbitrage. If there is any deviation, risk free arbitrage profits can be earned by simultaneously:

- buying the underlying asset (if the call is higher priced) or short selling the underlying asset (if the put is higher priced) by;
- selling the option with higher premium; and
- buying the option with lower premium.

Obviously only one of the two (call or put) will be exercised depending on the price. The purchase or sale of the underlying asset cancels this out leaving the difference in premium as profit. For strike prices other than at-the-money,

2 H. R. Stoll, 'The Relationship between Put and Call Option Prices,' *Journal of Finance*, December, 1969.

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the put and call premia are not equal due to the differing implications of the gap between the two prices. However, even in such cases it has been shown that there is predictable relationship between put and call premia for the same strike price which is explained below.

To understand PCP in a simple and relatively non-mathematical way, the reader should work through Example 11.2.

Example 11.2

Consider two portfolios of investment. One portfolio (Portfolio 1) consists of two elements:-

- *A European call option on 100 oz. of gold with a strike price of $1,600 and maturity date of 30 September.*
- *An amount of cash equal to the strike price of the option, viz., $1,600.*[3]

The second portfolio (Portfolio 2) comprises two elements:

- *A European put option on 100 oz. of gold at the same strike price ($1,600) and maturity date (30 September)*
- *Equal quantity of underlying (in this case Gold equivalent in value to 100 oz.*[4]*) at the current price, say, $1,570.*

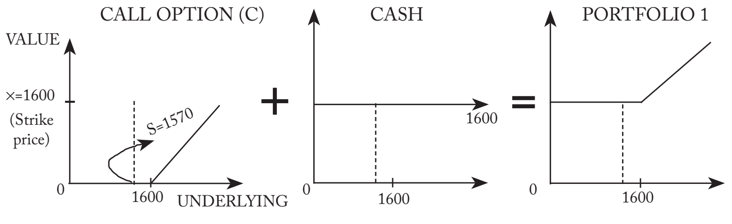
In portfolio 1, when the gold price falls, the value of the call option will also fall but the cash will remain intact. Alternatively, if the gold price rises, the call option will also rise in value while the cash remains unchanged.

In portfolio 2, when the price of gold falls, the put option will appreciate while the value of the gold (underlying) would fall. If the gold price rises, the put option declines in value but the underlying (gold) appreciates. These payoffs are shown in Figure 11.1:

Portfolio 1 = Call + Cash, where Cash amount = Call strike price

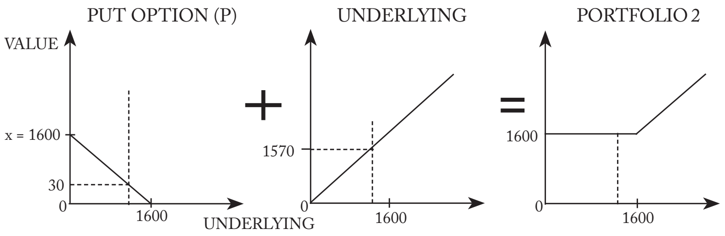
Portfolio 2 = Put + Underlying asset

***Figure 11.1:** Pay off diagrams for PCP*



3 For simplicity, interest is ignored at this stage or taken as zero (incidentally, in recessionary times this has sometimes actually been close to the truth—e.g., in Japan or lately in the United States as well.)

4 Storage cost of gold is assumed to be zero.



Note: Figure not to scale

Note that the combined pay off diagram for both portfolios in Example 11.3 is identical. From this it will be clear that the two portfolios are identical in terms of their payoff structure, i.e., (Call + Cash) has the same pay off as (Put + Stock) i.e.,

C+X=P+S

Where,

C = call premium

P = put premium

S = value of the underlying security or asset

X = exercise price (also known as strike price) for both the call and the put option

This (Call premium plus exercise price= Put option premium for the same exercise price plus value of the underlying security) is the PCP relationship when the interest rate is assumed to be nil. If there were a difference in the pay off, then it would be advantageous for an investor to do arbitrage, e.g., buy the cheaper portfolio, simultaneously sell the costlier portfolio and thereby earn a risk free profit. In liquid and well-traded markets any such opportunity would lead to quick and massive arbitrage until the parity is restored. For example, if puts are under-priced, traders would buy puts until the premium on puts increases and the parity is reached. *Hence, normally the two portfolios should have an identical value viz., should be at parity*.

In Example 11.2, the interest rate was assumed to be nil. If interest rates are taken into account, then the amount of cash required in portfolio 1 of Example 11.2 has to be adjusted so that with interest it would yield the strike price on the maturity date. Instead of an amount X, a smaller amount is needed. Accounting for this, mathematically the PCP relationship becomes:

$C + Xe^{-rt} = P + S$

Or

$P = C + Xe^{-rt} - S$

Where,

C = call premium

e= 2.71828 (Euler's number or exponential constant)

P = put premium

S= value of the underlying security or asset

X= exercise price (also known as strike price)

r= risk-free interest rate

PCP for ATM European options and American options

It may be noted that a European call is in-the-money when $S > Xe^{-rt}$ and a European put is in-the-money when $S < Xe^{-rt}$. In the special case of an at-the-money European option, the current market price is equal to the discounted value of the strike price, i.e., $S = Xe^{-rt}$.

We know that,

$P = C + Xe^{-rt} - S$

But $S = Xe^{-rt}$ Substituting S for Xe^{-rt},

$P = C + S - S$

i.e., $P = C$.

Thus, a put and a call have identical premia when the option is ATM.

For American options, the term Xe^{-rt} simply becomes X. Even for European options, Xe^{-rt} can be replaced by X for approximate calculations in respect of short term options when interest rates are very low because the difference between X and Xe^{-rt} is small.

The PCP can be used to derive approximate prices for calls or puts if the price of puts or calls (respectively) is known.

Example 11.3

The current stock price is ₹ 730 for Company XYZ, and the risk-free interest rate is 7.5 percent. A two month (European) put option with an exercise price of ₹ 500 has a price (premium) of ₹ 10, but due to low liquidity, there was no listed quote for the two month, ₹ 500 call. What is a good estimate of its value?

Rearranging the call-put parity equation,

$C = P + S - Xe^{-rt}$

Call value = 10 + 730 – 500(e–0.0125) = 740–493.8 [rt = (7.5/100)*(2/12)=0.0125]

= ₹ 246.2

Options pricing models

Over the years, several mathematical formulae have been evolved for calculating the composite (intrinsic plus time) value of options. Explaining the derivation of these formulae or how they work would be beyond the scope of this book, which explicitly aims to provide a non-mathematical approach. In practice, market participants need not do these calculations themselves as they can be programmed into a computer. However, a basic conceptual understanding of the methods is important for sellers and writers, to prevent mispricing, and for buyers in assessing the correctness of a given price. Accordingly, a short introduction to the main formulae is provided so that readers are aware of the most important techniques in use and as a basis for further reading.

Black-Scholes option pricing model

The most important options pricing model is known as the Black-Scholes option-pricing model.[5] This model (which was evolved by the economists Fischer Black and Myron Scholes) gives a formula by which the premium can be worked out.

The value of a European call option on shares under the Black-Scholes option pricing model is:

$c = SN(d_1) - Ke^{-rT} N(d_2)$

where,

c = European call premium

S = current market price of underlying asset or security

T = time left till maturity

K = strike price

$N(d_1)$, $N(d_2)$= cumulative normal distribution function of d_1 and d_2 respectively.

e = 2.71828 (Euler's number or exponential constant)

$d_1 = [\ln (S/K) + (r + s^2/2)T] \div [s\sqrt{T}]$

$d_2 = d_1 - s\sqrt{T}$

Ln = natural logarithm

5 F. Black and M. J. Scholes, 'The Pricing of Options and Corporate Liabilities', *Journal of Political Economy*, May, 1973. This seminal paper became the starting point for a whole body of literature on options pricing.

s = standard deviation of price changes of the underlying (i.e., volatility)

r = risk-free interest rate

This basic formula can be adapted to price European puts, for which the formula is:

$p = Xe^{-rT} N(-d_2) - SN(d_1)$,

where, p is the put premium.

There are adaptations of the Black-Scholes formula to price options on futures, American options, barrier options, commodity options and currency options (where there are two interest rates to be considered), etc. For some of these, the formulae are only approximate.

Binomial distribution or binomial options pricing model

This model is an alternative to Black-Scholes that is considered more accurate when dealing with options that may be exercised over a period of time rather than on a single date. It is often used to price American and Bermudan options, particularly for dividend-paying stocks. This alternative is only suitable when path dependence is not an issue, that is when it does not matter what path a security took to reach its present price or condition (see below).

Monte Carlo method

The Monte Carlo method[6] is a method which uses computer-generated algorithms that rely on a huge number of repeated random samples to compute the final results of various scenarios and hence generate a distribution. The sheer size of the iterations provides fairly robust results since all possible scenarios are tested. The disadvantage is that the sheer volume of computation and computational time involved makes this impractical for use on a real-time basis for actively traded markets. It is primarily used for one-off valuations of real or financial options (as in Example 11.4 below) rather than for repeated use in a trading desk.

The Monte Carlo method is often used for valuation of certain kinds of

6 The physicists John von Neumann, Stanislaw Ulam and Nichols Metropolis first used the Monte Carlo method in the 1940s while they were working on the nuclear weapon project (Manhattan Project) in the Los Alamos National Laboratory. The method was named after the Monte Carlo casinos.

financial options, (complex options involving multiple parameters including Asian options) and for real options. This method is particularly suited for situations involving 'path dependency'.

Example 11.4

A bank has a portfolio of fixed rate mortgages with different maturities and interest rates. The mortgages contain a pre-payment option which allows the borrower to pre-pay the loan. For purposes of securitising and selling this portfolio as a method of refinance, the Bank needs to be able to value the cost of the 'pre-payment option'. The option is more likely to be exercised when market rates of interest go down. Whether the options would be exercised by borrowers is 'path-dependent', i.e., the outcome depends on the exact path or sequence of events. If the same events happen but in a different sequence, the outcome may be different. (For instance, if interest rates rise and then fall after a few years, the outcome will be beneficial to the bank vis-à-vis a situation where they first fall for a few years and then rise, even if the average interest rate is the same in both cases.) This can be modelled with the Monte Carlo method and a price of the mortgage portfolio can thus be calculated. Then the calculation can be repeated without the pre-payment option. The difference in the portfolio values in the two cases represents the value of the pre-payment option in the mortgage.

Estimating volatility

Earlier, when describing the Black-Scholes option pricing model, we saw that one of the terms is: s = standard deviation of price changes of the underlying.

This is also known as volatility. Volatility is thus clearly a crucial input to the Black-Scholes model. The other inputs – risk-free interest rate, time to expiry, strike price, underlying price – are all known or easily discernible. Theoretically the volatility to be used is the future volatility but this is obviously not known. The question arises as to the best way of estimating the future volatility, based on extrapolating the past. There are various possibilities which have advantages and disadvantages.

Current volatility

The 'current' volatility – volatility in say the last trading day – is easy to obtain and reflects the current situation but is subject to large fluctuations (i.e., the volatility itself is volatile!). Another alternative is the average daily volatility of a longer period - say the last month. Even this can sometimes exhibit a large fluctuation when an exceptionally volatile trading day falls out of the sample.

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Another alternative is to take a weighted-average of the volatilities of a specified number of trading days, with a higher weight being given to the 'nearer' days (so long as the total of the probabilities add up to one).

Historical average volatility over a long period can also be combined with the weighted-average probability of the last few days (such that the sum of the weight of the historical average and all the different weights for volatility of the last N days add up to one). Depending on the parameters used, adding a historical average to the current weighted average can imply a mean-reverting volatility model. ('Mean reversion' is an assumption or belief that a variable will eventually revert to its historical mean and is often empirically borne out. For instance, if the price-earnings ratio of the Indian stock market has historically been at, say, 15 and is currently at 20, mean reversion would imply that the ratio will be expected to fall and come back to its historic level.) Such models for estimating volatility are various versions of the 'generalised autoregressive conditional heteroskedasticity' (GARCH) model.

Implied volatility

Implied volatility (i.v.) is the volatility implicit in the market prices of traded options according to a certain valuation model (like Black-Scholes). Instead of trying to estimate volatility based on historical data or some other method, the volatility becomes the unknown in the equation in which all other parameters (including put and call prices) are known. The volatility that emerges from the pricing model is the 'implied volatility', which (if it had been used in the formula) would have resulted in the current price structure.

Of course, to estimate volatility for a stock option's 'true value' one cannot use the volatility implied by the current trading price of that stock's existing options, as that would be assuming that the stock is fairly valued, and the model would simply return the market price! It would be an exercise is tautology. However, implied volatility can be used for valuing other kinds of options – employee stock options, over-the-counter exotic options, and so on. If traded options (long term or short term) exist comparable to ESOPs then a company can use the i.v., instead of estimating volatility from scratch to know the true cost (for accounting and other purposes) of these employee incentives.

Example 11.5

A NIFTY 5,000 call trades for ₹ 200 for a certain month's expiry date. ₹ 200 as call premium would be returned by the Black Scholes value only if the input parameter of volatility is 23 per cent annualised. The i.v., of this call option is 23 per cent.

The problem with using i.v., to estimate volatility is the assumption of 'market knows best' or in more technical terms, that a very strong form of market efficiency holds true. Experience has demonstrated that this assumption is often invalid.

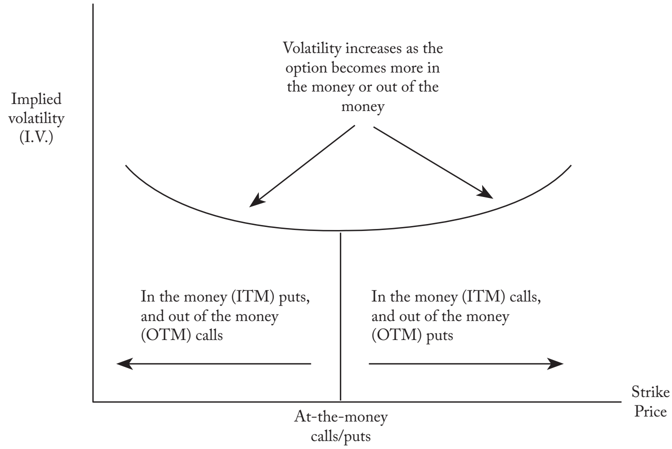
Moreover, if an investor wants to do 'volatility arbitrage' or 'vol arb' (statistical arbitrage[7] on the volatility – see later in this chapter), he obviously cannot rely directly on implied volatility because that would be a tautological error. If the i.v. of some option is 23 per cent and we assume that to be the true volatility, then there is no scope for profitable statistical arbitrage trading left, because the current market price itself gave us that number of 23 per cent! However, having the i.v. of similar options can be useful in 'vol arb' too: one could look for unusual patterns in the way the i.v., changes at different strike prices or different maturities and then trade on the basis that the i.v. will return to its 'normal' pattern. (The assumption that a pattern will revert to its previous average or mean is known as 'mean reversion'.)

Skews and smiles

On the basis of the conventional option pricing models, implied volatility should be the same across various strike prices. In other words, a graph of the volatility of a particular option at different strike prices should be a straight line parallel to the x-axis. However, this is rarely the case. Currency options are an example where it is often found that volatility-as-a-function-of-strike-price displays a 'smile' or a flat U-shaped structure (see Figure 11.2 below). This means that market participants do not expect currency movements to be 'normal', instead they expect the distribution to have a higher frequency or probability of more extreme currency movements (up or down) compared to movements suggested by a normal distribution. The presence of a 'smile' gives rise to an inference that the distribution of currency fluctuations is leptokurtic (that is, having thicker tails than a normal distribution). Non-normality strikes at the heart of options pricing models.

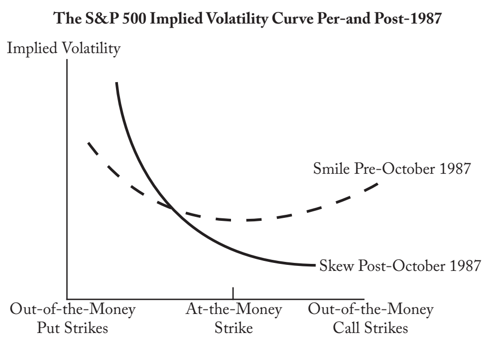
7 Note that this type of 'arbitrage' is not riskless.

Figure 11.2: Implied volatilities in a 'smile' pattern



Equity options on the other hand often exhibit a **skew** structure (see Figure 11.3 below which shows how the 'smile' converted into a skew for US equity markets after the 1987 crash); this implies that market participants feel that the probability of really big down moves is higher than the probability of really big up moves (beyond 2 or 3 standard deviations).

Figure 11.3: Implied volatility showing a skew



Source: CBOE (Chicago Board Options Exchange).

Another reason for the volatility structure to be skewed (instead of flat or 'smiling') could be the existence of portfolio insurance. Large institutions often

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take 'portfolio insurance' against downward price moves through the purchase of deep OTM put options, to protect themselves against large crashes. In so far as the second factor (i.e., skew due to need for insurance) dominates the first factor (expectation of more frequent down moves), there is scope for statistical and/or volatility arbitrage to make the structure more symmetrical – that is, by buying options at strike prices with relatively lower volatility, and selling those with higher volatility.

Risks in using option pricing models

The Black and Scholes Model owes its popularity to the fact that unlike many theoretical models, it can be applied to real life on the basis of observable statistics. The volatility of the underlying asset can be estimated from data on past price behaviour. Every other variable in the formula is directly observable. Over the years, several refinements to the model have been made on the basis of advanced theoretical research using complicated mathematical techniques. To a large extent, option writers base their price quotes on one or other version of the model (which may include modified versions involving the Monte Carlo, Binomial etc. methods outlined above). Theoretically, the use of the model is expected to yield a price for each option that exactly compensates the writer for the expected risks over a period of time, so that on the average, taking gains with losses, the writer incurs no net loss. (The actual cost to the buyer would of course include a margin for the writer.)

It is however crucial for the reader to understand that, notwithstanding the apparent precision of the Black-Scholes formula, option pricing is not an exact science. The Achilles' heel of the formula is the fact that while the volatility in the theoretical model is the volatility over the (future) period covered by the option, the figure of volatility actually 'plugged in' to the model for real-life calculations is based on a study of the past. In a sense, it is like driving a car by looking backwards. If future volatility mirrors or closely follows past volatility, the model gives good results. If, however, the volatility undergoes a big change, due to an unexpected fundamental change in the underlying market (say), the formula can produce inaccurate results causing losses to writers. Also certain assumptions are used in the derivation of the formula, which may not always hold in practice. The existence of smiles and skews is an example of this. Therefore, while formulae and models are useful guides to pricing, they should not be taken as fool-proof rules.

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Option price sensitivity and option Greeks

As seen in earlier chapters, the option price or premium depends on the price of the underlying, the interest rate, the time left to expiration, and the volatility of the underlying.

A change in any of these factors will lead to a change in option premia. The *sensitivity or rate of change of the option premium due to a change in these variables* is a parameter that options market participants monitor closely. A distinctive letter of the Greek alphabet denotes the sensitivity of the option premium to each of these factors. These are therefore called 'option Greeks'. The option Greeks are listed in Table 11.1 below:

Table 11.1: Option price sensitivities

Rate of change of option premium due to change in	..... is known as:
Price of the underlying	Delta
Time left to expiration	Theta
Volatility	Vega
Interest rate	Rho

Source: CBOE.

The delta itself changes in value at different prices. When an option is deeply OTM, a given change in the price of the underlying produces only a small change in the option premium; thus the delta is small. As the underlying price moves closer to the strike price, the option premium becomes more sensitive to changes in the price of the underlying. For instance, when the price of a share changes from ₹ 100 to ₹ 101 (i.e. 1 per cent), the premium on an ₹ 200 call may change by only 0.05 per cent. When the price of the share changes from ₹ 175 to ₹ 176.75 (1 per cent) with other parameters unchanged, the option premium on the same ₹ 200 call might change by 0.4 per cent. Thus, the delta itself has changed. The delta for deeply out-of-money options will be close to zero, for ATM options close to 0.5, and for deeply in-the money options, close to 1. Another Greek letter, gamma, is used to denote the rate of change of the delta. Mathematically speaking, delta, theta, vega and rho are the first derivatives[8] of the option premium with respect to underlying price, time, underlying volatility and interest rate, while gamma is the second derivative of the option premium with respect to underlying price.

8 The word derivative is used here in a mathematical sense and hence a different meaning from the rest of the book.

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